

# Leonardo da Vinci and Mathematics

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## Abstract

This paper deals with Leonardo da Vinci and mathematics. Leonardo was an incredible artist and inventor, which through the use of mathematics only made his projects even better. Leonardo da Vinci is a great mathematician, because he knew the difference between spiral vortices and eddies, understood the “golden ratio,” and invented numerous machines and devices.

## 1 Introduction

Leonardo da Vinci (1452-1519), was the ultimate Renaissance man. Leonardo was born in the hamlet of Anchiano, Italy. His father, Ser Piero da Vinci, was a notary, while his mother Caterina was a peasant. Leonardo was born illegitimate and in the 15th Century this was horrible. These children were pariahs; they were ridiculed and shunned by all. Leonardo was prohibited from attending any university. Leonardo was mainly raised by his grandparents and his Uncle Francesco. After the death of his grandparents, Leonardo moved to Florence with his father Piero, and began his apprenticeship with Andrea del Verroccio, a great Florentine sculptor and painter. (Verroccio was taught by Donatello, and had many successful students including Leonardo, Perguino, and Botticelli.) Although Leonardo stayed with Verroccio for over ten years, within six he was already certified a master painter, and a member of the painters’ guild, [1].

As a painter, Leonardo da Vinci was magnificent. There are only thirteen paintings accepted as being Leonardo’s and all are masterpieces. The “Mona Lisa,” c.1505-1514, is the only painting Leonardo claimed was a masterpiece. The “Virgin of the Rocks,” c.1483-1486, was his second masterpiece and is now located in the Louvre, Paris. The “Last Supper,” c.1495-1497, is in the end wall of the Convent Sta Maria delle Grazie, in Milan. It is Leonardo’s only large-scale work and encompasses the Golden Ratio. It is because of these proportions that an optical illusion is created: first, the wall that the “Last Supper”

is painted on looks like a dining hall instead of a wall with a painting on it, and secondly, regardless of position in front of this painting, the viewer is still drawn to the center of the painting, [3].

Although Leonardo is known as a famous artist and sculptor, not a single sculpture exists known to be made by him! Leonardo's fame comes from an incomplete project of a bronze horse that was never cast. The project lasted over ten years and was supposed to be a statue of Duke Francesco mounted on a horse, however Leonardo never started on the Duke. Leonardo never made any drawings or plans for the Duke, because he was so enthralled with the beauty of the horse. He also built a full scale model out of clay, that was over twenty-three feet tall, and he left detailed drawings of the iron framework and detailed drawings and instructions on how to transport and cast the horse. France was threatening Milan with attack, so they built a cannon instead of using the bronze for the statue, [2].

Leonardo is credited with over 300 inventions. One of Leonardo's duties as artist of the court, was to design the settings and scenery for festivities, such as plays and pageants. It was because of the theatre that Leonardo's genius as an engineer was more widespread. Leonardo created a machine to increasing volume during performances, to wow the audience. He also created a self-regulating spit, that, depending on how hot the flame was, it would rotate faster or slower, [2].

## 2 Mathematician

In 1490, Leonardo da Vinci was sent to Pavia to inspect the structural integrity of a cathedral. While working there, he discovered an enormous library in the Visconti Castle. There he met Fazio Cardano, a mathematician from the University of Pavia, who specialized in geometry and geometrical optics. Leonardo's discussions with Cardano piqued his interests in mathematics, and when he returned to Milan he began two new Notebooks, Manuscripts A and C; these notebooks dealt with problems involving weights, force, and movement. A few years later (1496), Leonardo met Luca Pacioli, a native Tuscan like Leonardo, and a mathematician. Pacioli helped Leonardo through the use of Euclid's *Elements*. Shortly thereafter, Leonardo and Pacioli put a book together, *De divina proportione*, written by Pacioli and illustrated by Leonardo. The book contained over sixty illustrations by Leonardo da Vinci, including drawings of the five regular polyhedra, along with a widespread overview of the "divine proportion" and its role in anatomy and architecture, [2].

## 2.1 Phi ( $\Phi$ )

Phi ( $\Phi$ ), which is approximately 1.618, is also known as the “Golden Ratio.” This extraordinary number shows up everywhere, from paintings, to architecture, to the pyramids, even the bible!

“For of three magnitudes, if the greatest (AB) is to the mean (CB) as the mean (CB) is to the least (AC), they therefore all shall be one [4].”

Thus, AB is to CB as CB is to AC or 1:1.618...

$\Phi$  can also be obtained through the Fibonacci series: 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233... By dividing a number in the series by the number that precedes it, that ratio gets closer to  $\Phi$ . Leonardo was fascinated by this number and used it throughout his works.

$\Phi$  can also be obtained algebraically, [6].

$$n^2 - n^1 - n^0 = 0$$

$$n^2 - n - 1 = 0$$

$$n^2 = n + 1$$

$$(1 + \sqrt{n})/2$$

$\Phi$  is obtained when  $n=5$ , or

$$(1 + \sqrt{5})/2$$

$\Phi$  is evident in Leonardo da Vinci's, “The Last Supper,” see Figure 1. Jesus is the center of the painting. The ratio of the distance from the line above Jesus' head, to the floor, and from the line to the top is  $\Phi$ . Also there is a rectangle created around Jesus. From left to right, and from right to left, using this rectangle is  $\Phi$ .

$\Phi$  is also apparent in the anatomy of man. If measured from top of head to top of eye, see Figure 2, and from top of eye to chin, the ratio is close to  $\Phi$ . Also, top of head to navel, and from navel to bottom of feet is close to  $\Phi$ , [5].

Plants that are formed in spirals, such as pineapples, pine cones, and sunflowers demonstrate the “Golden Ratio.” In sunflowers, going clockwise, see Figure 3, there are 55 spirals and 89 spirals going counterclockwise. Both numbers are in the Fibonacci series, and their ratio is approximately  $\Phi$ , [5].

Figure 1: “The Last Supper,” [5]

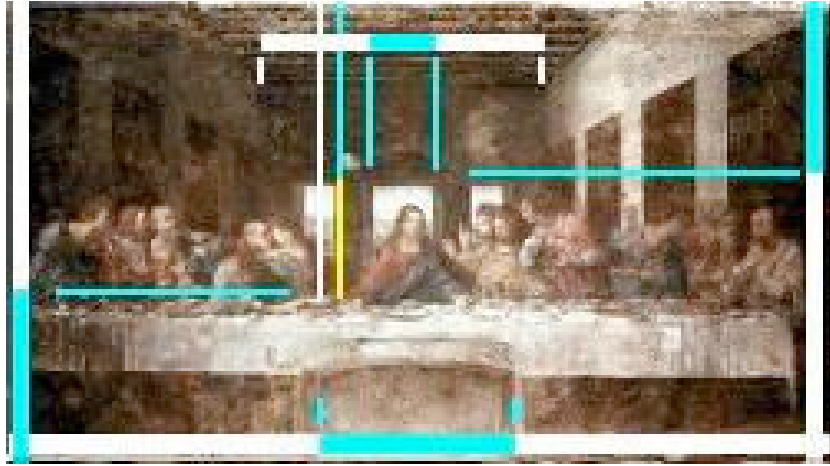
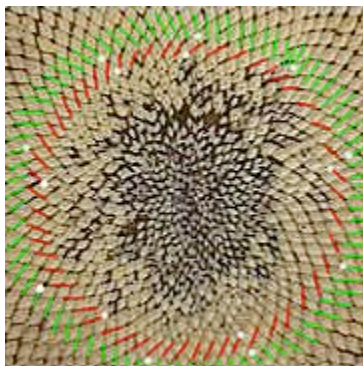


Figure 2: Head Proportions, [5]



Figure 3: Sunflower Spirals, [5]



## 2.2 Spiral Vortices and Eddies

Leonardo da Vinci was fascinated by water and spent endless hours observing the flows of rivers and tides. He produced hundreds of drawings of rivers and lakes with different sized whirlpools depicted throughout. In many of his paintings, a river or lake serves as the back drop. Leonardo knew the difference between spiral vortices, which form a tornado-like funnel at the center, and eddies, where the water rotates as a whole on the surface, [2].

## 2.3 Machines and Inventions

Leonardo da Vinci was intrigued by the possibility of human flight. With his knowledge of the human anatomy, the anatomy of birds, and mechanical engineering, Leonardo made designs for the “Flying Ship.” The pilot was to be stationed in the center and by pushing two pedals with his feet and at the same time turning two handles with his hands, he would, in theory, be able to generate enough force to loft off the ground. Leonardo made other designs where the pilot is in a horizontal position, but he never created a model. Leonardo realized that flight by flapping mechanical wings would not be possible because the machines would be too heavy to be lifted by man. So instead, he redirected his thought and made a plan for a flying machine with fixed wings. This machine known as “Leonardo’s Glider,” was built and tested by engineers from England, and actually worked, [2]!

Leonardo da Vinci was keenly aware of friction and its effects on moveable parts in machines. Through observation and experiment, Leonardo correctly identified that friction was determined by the roughness of the surface (frictional coefficient), the inclination/slope

of the surface, and the weight of the object. To combat friction, Leonardo invented many devices, including a rotary ball bearing. The rotary ball bearing was made of wood, with balls trapped in place by eight concaved spindles.

### 3 Conclusion

Leonardo da Vinci was a superb artist and inventor. He was also a gifted and knowledgeable mathematician. Leonardo understood the principles involved with the “golden ratio,” knew the difference between eddies and spiral vortices, and created numerous machines and devices.

### References

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